The Effects of Labour Market Policies When There is a Loss of Skill During Unemployment.

by

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This version, 16th December 1995

Abstract

In this paper, we analyse the labour market using a matching model. In our labour market, there are two types of workers: primary workers; and secondary workers. Primary workers are those workers who, when in employment, are fully productive and, when in unemployment, have a maximum search intensity. Secondary workers, on the other hand, may be less than fully productive when employed. In addition, they may have a lower search intensity than primary workers when unemployed. A primary worker becomes a secondary worker by first spending a length of time in unemployment. Thus the event of an unemployed primary worker becoming a secondary worker is duration dependent. An unemployed secondary worker can become a primary worker by either first being employed as a secondary worker or by taking a place on a labour market programme. However, in this model we allow for the possibility that taking a job or a place on a labour market programme may not guarantee that the worker will become a primary worker. In this paper, labour market programmes are directed at secondary workers in unemployment. The general result of this *modus operandi* is ambiguous. The proportion of primary workers, the proportion of secondary workers, and the rate of total unemployment can all either increase, decrease, or remain unchanged, when labour market programmes are used more intensively. The same is true of total production in the economy.

1 Introduction

* Useful comments from Geezer Butler, Bertil Holmlund and Arthur van Soest are acknowledged.

Until recently, many observers hailed Sweden's ability to maintain low levels of unemployment as an outstanding success. This was especially pertinent in light of the ravaging unemployment which Great Britain and much of Europe was experiencing. Often, this success was attributed to Sweden's extensive use of active labour market policies. Miller (1991) states that "whilst most OECD countries spend only a fraction of 1% of GNP on active labour market policies, Sweden spends between two and three percent". Many observers, seeing Sweden's prevalent use of active labour market policies together with its low rate of unemployment, viewed the former as a cause of the latter. There appears, however, to be rather little empirical evidence to support such an opinion. Indeed, in the last few years, Sweden has itself experienced unemployment rates of mainstream European magnitude, despite the continued usage of active labour market policies. For example, in 1988 the unemployment rate was 1.6% whilst the percentage of the labour force in labour market programmes was 1.3%. Yet in 1993, unemployment had reached a post-war high of 8.2% whilst the percentage of the labour force on labour market programmes had reached around 4%. Thus it is clear to see that there has been no let up in Sweden's usage of labour market programmes.

A notable feature of unemployment is the dispersion of its durations. As Layard, Nickell & Jackman (1991) point out, "between 1979 and 1986, the proportion of unemployed who had been out of work for over a year rose from around twenty to around forty percent in Britain". Thus we can see quite clearly that one problem associated with unemployment is not merely its level but also its duration. Furthermore, the widely varying duration rates of unemployment often have little to do with the initial characteristics of the unemployed individual. Jackman & Layard (1991), for example, find little evidence of the existence of heterogeneity as a causal factor of unemployment duration. They further find strong evidence for the dependency of the rate of exit from unemployment on the length of duration of unemployment: "The proportion of unemployed people who leave unemployment within a given time period is much lower for those who have been unemployed for longer durations. For example, in Britain in early 1984 the proportion was four percent per quarter for men who had been unemployed for over four years, compared with forty percent for men unemployed under three months". Thus any matching model of the labour market will be more realistic if it encompasses some form of distinction between the rates of exit into employment from long- and short-term unemployment.

Whilst there is evidence of differing exit rates from unemployment depending on duration, there is also a possibility that longer unemployment durations result in a loss of human capital for the worker. Just as the acquisition of skills, i.e. the formation of human capital, is a positive process, so is the depletion of skills a negative process. When a worker is unemployed, especially for long durations, skills become rusty and the worker's productive potential declines. Layard, Nickell, & Jackman, Jackman & Layard, and Pissarides (1992) all point to this possibility. If this is the case then it will certainly be worthwhile to incorporate the possibility of skill loss resulting from unemployment duration in our model.

With regard to reduced search intensity as a result of unemployment duration, there are two strong reasons as to why this may occur. Firstly, a worker may become discouraged from searching for work due to the lack of success to date. Having searched for work but had no success in obtaining any, the worker may begin to feel that search is simply not worth the effort. They therefore curtail the amount of effort they expend on search. Secondly, search intensity may be reduced due to the fact that search costs money and those who have been out of employment for a given length of time may be unable to afford the level of search which they would wish to choose if unconstrained by their budget.

The OECD Employment Outlook (1995) points to much evidence on workers reducing their search intensity significantly with unemployment duration, though still remaining within the labour force with a limited search intensity. This is evidenced in a multitude of countries. Layard, Nickell, & Jackman looking at evidence for Great Britain, find that there is some evidence of a decline in the amount of time spent in search as a result of duration of unemployment. They find further that the amount of money an unemployed worker spends on search activity declines heavily with duration of unemployment. This can be seen in the following table:

Table 1

British Male Long-Spell Unemployed Workers, 1978/79.

Hours of search per week	6 weeks duration	12 months duration
Up to 5 hours per week	54	64
6-9 hours per week	16	17
10 or more hours per week	30	20
	100	100
Money spent searching/week	6 weeks duration	12 months duration
Nothing	26	42
Under £1	35	34
£1 - £3	26	18
£3 or more	13	6
	100	100

Source: Layard, Nickell, and Jackman (1991), Page 236. These figures, which are percentages, refer to long-spell unemployed workers. By this, they mean those workers who remained out of work after one year's unemployment. These workers were surveyed after 6 weeks of unemployment, and later after one year of unemployment.

Thus we see that there are strong reasons as to why workers may wish to reduce their level of search intensity as the period of time they spend as unemployed increases. Furthermore, we see that there does exist empirical evidence of reduced search intensity as a result of duration of unemployment. We feel, therefore, that to incorporate this possibility into our model adds significantly to our analysis.

Pissarides (1992) looks at skill loss within an overlapping generations framework. In an environment where firms are unable to discriminate *ex ante* between workers of different skill levels, he finds that a one-period negative shock to employment can persist for a long time after the duration of the shock and the maximum duration of unemployment. The negative shock leads to a fall in hiring which in turn leads to an increase in the duration of unemployment. This increased duration leads to a loss of skill meaning that workers become less attractive to firms. As a result, fewer jobs come onto the market next period. Thus the unemployment duration of the new cohort of unemployed is also above the trend value, even if all of the unemployed from the last period have exited from unemployment.

Another attempt at explaining the persistence result of a negative shock to employment is that of Diego (1994). Diego again uses an overlapping generations framework with a matching structure to analyse the persistence of a shock. He finds that a fall in skill resulting from unemployment leads to the persistence of transitory shocks. This is despite the fact that firms are able *ex ante* to discriminate between different types of workers, unlike in the Pissarides paper. What drives the persistence result here, is a certain degree of complementarity in production between workers with different skill levels.

Whilst there is an abundance of literature on the usage of labour market policies, little is really mentioned as to *how* these labour market policies are actually carried out. In this paper, we look at a matching model where there is a single labour market but two types of workers: primary workers; and secondary workers. Primary workers exhibit maximum productivity and maximum search intensity. Secondary workers, on the other hand, exhibit either lower productivity, in relation to primary workers, and/or low search intensity. Primary workers move from the primary labour force to the secondary labour force by flowing from primary unemployment to secondary unemployment. Exit from the secondary sector to the primary sector, however, can occur through one of two ways: Firstly, a worker can exit from a secondary job into primary unemployment; and, secondly, a worker can leave secondary unemployment to gain a place on a labour market programme and from there exit into primary unemployment. The very fact that a worker has recently held a job or been on a labour market programme can rehabilitate them into the primary labour force. Indeed, the usage of labour market programmes in this model is limited to attempting to rehabilitate unemployed workers back into the primary labour force. This *modus operandi* is strongly motivated by such comments as that made by Jackman & Layard that “... if long-term unemployment can destroy human capital, it is more likely that work experience can rebuild it”.

It is important to note, however, that just because a secondary worker has taken a job or been on a labour market programme, this does not necessarily transform them into a primary worker. Thus in this paper, we allow for the possibility that a worker who has taken a secondary job or a place on a labour market programme may exit back into secondary unemployment rather than primary unemployment. How we model this will be seen later in this paper, but essentially the worker taking a secondary job or a place on a labour market programme can see it as a lottery. Whether the worker exits into the primary labour force or the secondary labour force is a matter of chance.

Three characteristics of this study distinguish it from other matching-model analyses. Firstly, it allows for the possibility of skill loss, as a result of unemployment duration. Thus the longer a worker stays in unemployment, the more likely is that worker to become less-than-fully productive. Secondly, it provides for the possibility that workers may become less likely to obtain a job, as a result of length of time in unemployment. This may be the result of the following: (a) Firms may prefer to search for primary workers in the labour market; and (b) secondary workers may search less intensively for a job than primary workers, due to being discouraged from their lack of success to date. Allowing for a lower possibility of a match occurring between a firm and an unemployed worker due to duration of unemployment has, however, also been analysed in a matching framework in Miller (1995b).

The final difference, that the participation on a labour market programme or the acceptance of a regular job may not transform a secondary worker into a primary worker, is a unique idea within the matching-model framework. It allows for the possibility that labour market programmes may not be designed correctly and that they may merely be a stop-gap between spells of secondary unemployment. Thus the idea that participation on a labour market programme will necessarily transform a worker from a secondary worker to a primary worker is relaxed in this paper. Similarly, the idea that the acceptance of a regular job by a secondary worker will transform that worker into a primary worker is also relaxed. The addition of these possibilities to the model make it a far more general model with which to analyse the labour market than otherwise.

In this paper, we undertake some simulations to see how the targeting of labour market programmes at those in secondary unemployment affects the labour market. From our simulations, we find that anything can happen. The proportion of the labour force who are primary workers and the proportion of the labour force who are secondary workers, can either increase, decrease, or remain unchanged This result, however, depends on how well labour market programmes transform secondary into primary workers in comparison with secondary employment. The effect on the total unemployment rate is ambiguous.

This paper takes the following format: In the next section, we make our model explicit, setting out the framework and assumptions from which our results are derived. In Section 3, we briefly review the comparative statics of the model. In Section 4, we undertake some simulations to gain an idea of the likely outcomes of using labour market programmes. In Section 4.1, we look at an example of a labour market where there is no skill loss, but search intensity is lower amongst secondary workers. In this example, secondary workers have a fifty percent chance of exiting into the primary labour force from either a regular job or a labour market programme. In Section 4.2, we analyse a labour market where both skill loss and lower search intensity are exhibited by secondary workers. If a secondary worker takes a regular job, they are guaranteed to exit from that job into the primary labour force. However, if they take a place on a labour market programme, then they only have a fifty percent chance of exiting into the primary labour force.

In Section 4.3, workers in the secondary labour force again exhibit lower productivity and lower search intensity; though this time, if they take a place on a labour market programme they are guaranteed to exit into the primary labour force, whilst if they take a regular job, they only have a fifty percent chance of exiting into the primary labour force. Section 4.4 shows a labour market similar to that analysed in Section 4.3 except that skill loss is even more pronounced. In our final simulation, in Section 4.5, we see an example of a labour market where skill loss is present though not any reduction in search intensity. In this example, a secondary worker who takes a regular job is guaranteed to exit into the primary labour force. A secondary worker who takes a place on a labour market programme, however, has only a ten percent chance of exiting into the primary labour force.

For all the simulations which we undertake, we start each analysis from a situation where there are no labour market programmes and proceed by increasing the flow out of secondary unemployment into programmes. By starting with a base run where labour market programmes are absent we are able to compare different intensities of the usage of such programmes with a situation where they are absent. Section 5 summarises the results from the preceding analysis and attempts to draw a conclusion.

2 The Model

The model which we consider in this paper is a matching model of the labour market where the matching of workers to jobs is both costly and time-consuming. Thus, in equilibrium, there is unemployment because a well-defined labour market does not exist. Firms do not immediately meet workers. Workers are matched to jobs by an aggregate matching function $H \equiv h(S,V)$, where S is the number of searchers and V is the number of vacancies. H is characterised by constant returns to scale technology and is increasing in both its arguments. Thus, should there be more searchers in the labour market, then there will be more matches taking place, firms finding it easier to find workers. Similarly, more vacancies will make it easier for a worker to find a vacancy and fill it. The number of searchers is equal to the number of primary workers in unemployment plus some proportion of the number of secondary workers in unemployment. Thus we have the identity $S \equiv U_1 + cU_2$, where U_1 and U_2 denote the number of primary and secondary workers in unemployment respectively and $0 < c \leq 1$ The motivation for allowing secondary unemployment to be weighted by c[1] which is able to take a value below unity, lies in the possibility that those in secondary unemployment do not search as intensively as those in primary unemployment.

In the economy which we analyse, the total labour force (L) is fixed. Thus we do not allow for flows into or out of the labour force. In this paper, we denote the rates of primary and secondary unemployment as $u_1 \equiv U_1/L$ and $u_2 \equiv U_2/L$ respectively. (The convention hereafter will be that all lower-case letters refer to the respective rates of the stocks concerned. Thus e_1, e_2, and r refer to the rate of employment of primary workers, the rate of employment of secondary workers, and the proportion of the labour force on labour market programmes, respectively.) The vacancy rate is denoted by $v \equiv V/L$. We let $\theta \equiv V/S$ represent labour market tightness. An increase in θ implies an increase in labour market tightness, and vice-versa. We further let $q \equiv H/V$ represent the rate at which vacant jobs are filled. Due to the constant returns to scale assumption we have

[1] We assume that $c \neq 0$, since equality implies that labour market programmes must always be in use. This would clearly be unreasonable, and would prevent us from analysing the difference between situations with and without labour market programmes.

$q(\theta) \equiv h(S/V,1) \equiv h(1/\theta,1)$, where $q'(\theta) < 0$. The flow of new hires into jobs is given as $H \equiv \alpha S$, where $\alpha \equiv (H/V)(V/S) \equiv q(\theta)\theta$. The function $\alpha(\theta)$ is increasing in its argument.

Since $H \equiv \alpha S \equiv \alpha(U_1 + cU_2)$, it is clear that $H/V \equiv \alpha U_1/V + \alpha cU_2/V$. The first term, $\alpha U_1/V$, is in fact the rate at which a vacant job is filled by a primary worker whilst the second term, $\alpha cU_2/V$, is the rate at which a vacant job is filled by a secondary worker. Denoting the rates at which vacant jobs become filled by primary and secondary workers as q_1 and q_2, respectively, allows us to write the following identity: $q \equiv q_1 + q_2$. Furthermore, we can view both q_1 and q_2 as some portion of q. By setting $q_1 \equiv \psi H/V$ and $q_2 \equiv (1-\psi)H/V$, we arrive at the following identities:

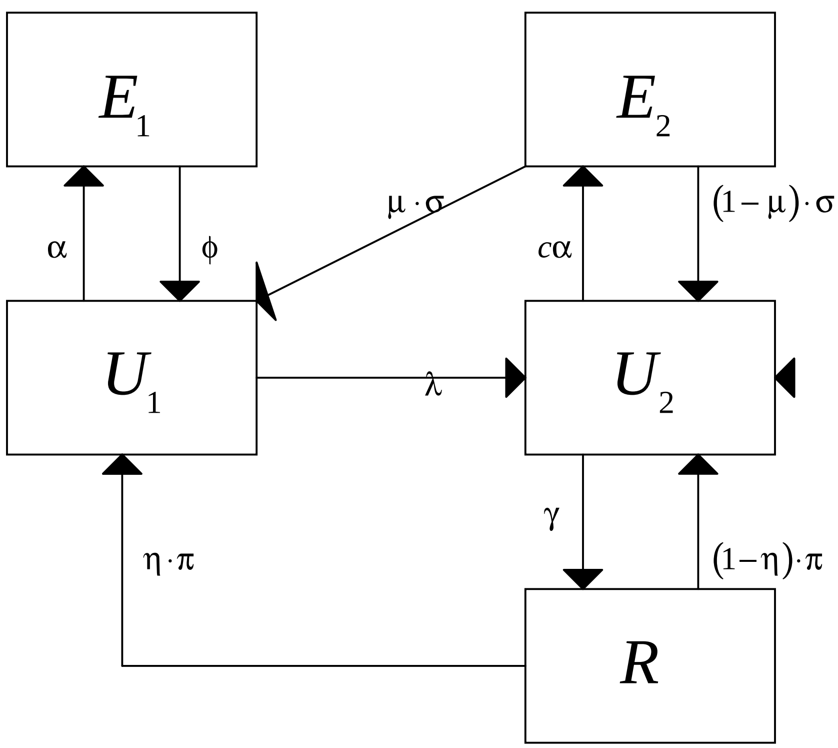
$$q_1 \equiv \left(\frac{U_1}{U_1 + cU_2}\right)q(\theta) \quad \text{and} \quad q_2 \equiv \left(\frac{cU_2}{U_1 + cU_2}\right)q(\theta)$$

We assume that regular job offers arrive according to a Poisson process. The arrival rate for a worker in U_1 is α, whilst the arrival rate for a worker in U_2 is $c\alpha$. Thus if $c < 1$, then the arrival rate of job offers to an unemployed secondary worker is lower than that of an unemployed primary worker. In our model, places on labour market programmes are only available to secondary workers in unemployment. Offers for placements on labour market programmes arrive according to a Poisson process with parameter γ. Notice that in this model there is no on-the-job search.

ϕ and σ are the exogenously given rates at which regular jobs held by primary and secondary workers break up, respectively. Placements on labour market programmes break up at a rate π which is government-determined. Since we consider labour market programmes to be temporary in this paper, we assume both that $\pi > \phi$ and that $\pi > \sigma$, i.e. that placements on labour market programmes break up at a faster rate than either of the regular employment categories. We assume that π is also bounded from above since labour market programmes are used to rehabilitate secondary workers back into the primary labour force. If the length of time spent on a labour market programme were too short, then the possibility that a labour market programme would to transform a secondary worker into a primary workers would be zero.

A worker will find themself in one of five possible labour force states: primary employment (E_1); secondary employment (E_2); primary unemployment; secondary unemployment; and on labour market programmes (R). A diagrammatic representation of our model is given in Figure 1, below. The boxes E_1, E_2, U_1, U_2, and R, refer to the stocks of primary employment, secondary employment, primary unemployment, secondary unemployment, and those on labour market programmes, respectively. The arrows represent the flows between the stocks. As can be seen, there is only one route of exit from the primary labour force to the secondary labour force: that of flowing from primary unemployment into secondary unemployment. However, there are two ways of leaving the secondary labour force to enter the primary labour force. The worker can either exit from secondary employment; or they can gain a place on a labour market programme and from there enter the primary labour force. Either way, the act of gaining either a secondary job or a placement on a labour market programme has the possibility of rehabilitating the worker into the primary labour force. Though this is not guaranteed.

Figure 1.



For a steady-state equilibrium, we require that the flows into a stock equal the flows out of the said stock. Thus we have the following equations:

[1] $\phi e_1 = \alpha u_1$

[2] $(\alpha + \lambda)u_1 = \phi e_1 + \mu\sigma e_2 + \eta\pi r$

[3] $\sigma e_2 = c\alpha u_2$

[4] $(c\alpha + \gamma)u_2 = (1-\mu)\sigma e_2 + \lambda u_1 + \pi(1-\eta)r$

[5] $\pi r = \gamma u_2$

(We also have the identity $1 \equiv e_1 + e_2 + u_1 + u_2 + r$.)

The above equations determine the various proportions of the stocks, given θ. (Note that we have $\alpha = \alpha(\theta)$.) To obtain the value of θ, we must look at how wages and vacancies are determined. Essentially there are two sides to the wage bargain: the firm's side; and the worker's. Firstly, we shall look at the firm's side.

All firms are small. Each firm has only one job which is either occupied or vacant. All vacancies are the same. They can be filled by either a primary worker or a secondary worker. In this model, firms are able to distinguish between primary and secondary workers. There is complete information in this model. Furthermore, there is no complementarity in production between primary and secondary workers. Thus, if we crowd out secondary employment, for example, this will not affect the productivity of those in primary employment.

A job filled by a primary worker has an expected present value to the firm of J_{o1}, whilst a job filled by a secondary worker has an expected present value to the firm of J_{o2}. A vacant job yields an expected present value to the firm of J_v. Letting δ represent the discount rate, y the constant marginal product of a primary worker, a a multiplicative term between zero and one to represent the possibility of lower productivity of a secondary worker due to their susceptibility to skill loss, and k the cost of maintaining a vacancy, then J_{o1}, J_{o2}, and J_v satisfy the following equations:

[6] $\delta J_{o1} = y - w_1 + \phi(J_v - J_{o1})$

[7] $\delta J_{o2} = ay - w_2 + \sigma(J_v - J_{o2})$

[8] $\delta J_v = -k + q_1(\cdot)(J_{o1} - J_v) + q_2(\cdot)(J_{o2} - J_v)$

(w_1 and w_2 refer to the wages of primary and secondary workers, respectively.)

As can be seen, a vacancy involves a cost per period of k and is turned into an occupied job either at the rate $q_1(\cdot)$ for a primary worker or $q_2(\cdot)$ for a secondary worker. Vacancies are kept open for as long as they yield a positive profit. Due to the small firm assumption, $J_V = 0$ in equilibrium. The value of a job occupied by a primary worker is found from [6] to be $J_{o1} = (y - w_1)/(\delta + \phi)$, and the value of a job occupied by a secondary worker is found from [7] to be $J_{o2} = (ay - w_2)/(\delta + \sigma)$. Substituting J_{o1} and J_{o2} into [8] yields the zero-profit condition for firms:

[9] $$(\delta + \phi)(\delta + \sigma)k = q_1(\cdot)(y - w_1)(\delta + \sigma) + q_2(\cdot)(ay - w_2)(\delta + \phi)$$

But why would a firm ever fill a vacancy with a secondary worker when there are primary workers in the labour market looking for employment? Quite simply, the reason for filling a job with a secondary worker is that the other option open to the firm is to wait for a primary worker to come along and fill the vacancy. The choice is not between filling the position with a secondary worker and filling the position with a primary worker, but rather between filling the vacancy with a secondary worker or leaving the vacancy unfilled. Thus the firm chooses to employ secondary workers.

Having discussed the firm's side of the wage bargain, we shall now discuss the worker's side of the story before making the bargaining scheme explicit. Since there are five possible labour-market states for a worker to find themself in, there are five possible value functions. We let Λ_{e_1}, Λ_{e_2}, Λ_{u_1}, Λ_{u_2}, and Λ_r represent the expected discounted lifetime income for workers in primary employment, secondary employment, primary unemployment, secondary unemployment, and labour market programmes, respectively. The value functions are as follows:

[10] $$\delta\Lambda_{e_1} = w_1 + \phi\left(\Lambda_{u_1} - \Lambda_{e_1}\right)$$

[11] $$\delta\Lambda_{u_1} = \rho_1 w + \alpha\left(\Lambda_{e_1} - \Lambda_{u_1}\right) + \lambda\left(\Lambda_{u_2} - \Lambda_{u_1}\right)$$

[12] $$\delta\Lambda_{e_2} = w_2 + \mu\sigma\left(\Lambda_{u_1} - \Lambda_{e_2}\right) + (1-\mu)\sigma\left(\Lambda_{u_2} - \Lambda_{e_2}\right)$$

[13] $$\delta\Lambda_{u_2} = \rho_2 w + c\alpha\left(\Lambda_{e_2} - \Lambda_{u_2}\right) + \gamma\left(\Lambda_r - \Lambda_{u_2}\right)$$

[14] $$\delta\Lambda_r = \rho_r w + \eta\pi\left(\Lambda_{u_1} - \Lambda_r\right) + (1-\eta)\pi\left(\Lambda_{u_2} - \Lambda_r\right)$$

(Where $w \equiv \left(e_1 \cdot w_1 + e_2 \cdot w_2\right) / \left(e_1 + e_2\right)$ is the average wage.)

As can be seen, benefits to primary workers in unemployment are linked to the average wage via the replacement ratio ρ_1, whilst unemployment benefits to secondary workers in unemployment and pay to those on labour market programmes are both linked to the average wage via ρ_2 and ρ_r respectively. All replacement ratios lie between zero and one. The model exhibits incentive compatibility in that the discounted expected lifetime income from holding a regular job is always higher than the value of being unemployed. Since we direct labour market programmes at those secondary workers in unemployment, we require that $\Lambda_r > \Lambda_{e_2}$. Notice that we do not require that $\Lambda_r > \Lambda_{u_1}$, since labour market programmes are targeted purely at those in secondary unemployment.

The wage of both primary and secondary workers is the result of a Nash bargain. For workers in the primary labour force, the Nash bargain in a particular firm i solves

$$\max_{w_{1_i}} \Omega_1(\cdot) = \left[\Lambda_{e_{1_i}}\left(w_{1_i}\right) - \Lambda_{u_1}\right]^{A}\left[J_{o1_i}\left(w_{1_i}\right) - J_v\right]^{1-A}$$

$$0 < A < 1$$

where Λ_{u_1} is the fall-back value for primary workers should they not gain regular employment. The outcome of this Nash bargain is the following wage equation:

[11] $$w_1 = y - \left[\frac{1-A}{A}\right](\delta + \phi)\left[\Lambda_{e_1} - \Lambda_{u_1}\right]$$

where the equilibrium conditions $w_{1_i} = w_1$ and $J_v = 0$ are imposed. For workers in the secondary labour force, the Nash bargain solves

$$\max_{w_{2_i}} \Omega_2(\cdot) = \left[\Lambda_{e_{2_i}}\left(w_{2_i}\right) - \Lambda_{u_2}\right]^{\mathrm{B}}\left[J_{o2_i}\left(w_{2_i}\right) - J_v\right]^{1-\mathrm{B}} \qquad 0 < \mathrm{B} < 1$$

where Λ_{u_2} is the fall-back value for secondary workers should they not gain regular employment This Nash bargain yields the following wage equation:

[12] $$w_2 = ay - \left[\frac{1-\mathrm{B}}{\mathrm{B}}\right](\delta + \sigma)\left[\Lambda_{e_2} - \Lambda_{u_2}\right]$$

where again the equilibrium conditions $w_{2_i} = w_2$ and $J_v = 0$ are imposed. As can be seen, any policy which reduces the discounted expected income difference $\Lambda_{e_j} - \Lambda_{u_j}$, $j = 1,2$, will increase w_j.

The complete model consists of sixteen equations, namely the flow-equilibrium conditions [1] - [5], the zero-profit condition [9], the value functions [10] - [14], and the two wage equations [11] and [12]. In addition, we have $\alpha = \alpha(\theta)$, $q_1 = \left[u_1/(u_1 + cu_2)\right]q(\theta)$, and $q_1 = \left[cu_2/(u_1 + cu_2)\right]q(\theta)$. These equations determine the endogenous variables: e_1, e_2, u_1, u_2, r, α, θ, q_1, q_2, Λ_{e_1}, Λ_{e_2}, Λ_{u_1}, Λ_{u_2}, Λ_r, w_1, and w_2.

3 Comparative Statics

In this section, we show the comparative statics of the model. Since a change in the policy parameter γ has both a direct effect on the size of a stock and an indirect effect on a stock through its influence on labour-market tightness θ, we show both the derivative of the stocks with respect to γ and the derivative of the stocks with respect to labour-market tightness θ. The comparative statics of the model are as follows: $\partial e_1/\partial\gamma \gtrless 0$; $\partial e_1/\partial\theta > 0$; $\partial e_2/\partial\gamma < 0$; $\partial e_2/\partial\theta \gtreqless 0$; $\partial u_1/\partial\gamma \gtrless 0$; $\partial u_1/\partial\theta \gtrless 0$; $\partial u_2/\partial\gamma < 0$; $\partial u_2/\partial\theta < 0$; $\partial r/\partial\gamma > 0$; $\partial r/\partial\theta < 0$.

The derivative of the various stocks with respect to the policy parameter γ, show the qualitative relationship between the stock concerned and the policy parameter when labour-market tightness θ is held constant. Also shown is the derivative of the various stocks with respect to θ, i.e., the indirect effect. As can be seen, the direct effect of an increase in the flow into labour market programmes is to increase e_1, u_1, and r, and to reduce e_2 and u_2. However, to gain the overall qualitative effect of an increase in γ on a stock's size, we must see what happens to θ and how this affects the stock. We find that we are unable to sign the relationship between γ and θ, i.e. $\theta_\gamma \gtreqless 0$[2]. Thus the proportion of each of these stocks can either increase, decrease, or remain unchanged. To see clearly what happens when the flow out of secondary unemployment and into labour market programmes is increased, we need to resort to some numerical simulations.

4 Some Simulations

In this section, we show examples of simulations which we have undertaken. These simulations include skill loss and lower search intensity as well as the possibility that taking a regular job or a place on a labour market programme may not transform the secondary worker into a primary worker. Each simulation starts from a base run without labour market programmes. From this base run, we increase the flow from secondary unemployment onto labour market programmes.

In our simulations, we also analyse how total production in the economy may be affected by using labour market programmes. Using the following definition of total production

$$TP \equiv e_1 y_1 + e_2 y_2 + r y_r - vk$$

where y_1, y_2, and y_r denote the production of a worker in e_1, e_2, and r, respectively, we give some idea as to how total production changes. In the following tables, TP_1 indicates total production for the economy where those on labour market programmes are totally unproductive, i.e. $TP_1 = e_1 y + e_2 a \cdot y - vk$. TP_2, on the other hand, gives a measure for total production in the

[2] This is difficult to prove analytically. However, simulations were undertaken which confirm that the sign of the relationship is ambiguous. Details available from the author on request.

economy where those on labour market programmes are as productive as those in secondary employment, i.e. $TP_2 = e_1 y + (e_2 + r)ay - vk$.

4.1 No Skill Loss but Lower Search Intensity. 50% Chance of Exiting into the Primary Labour Force from Secondary Employment and Labour Market Programmes.

In this section, we look at an example where there is no skill loss, i.e. a=1.0, but search intensity is lower amongst those in secondary unemployment. In this example, we let c=0.5. A secondary worker in either regular employment or on a labour market programme has a fifty percent chance of exiting into the primary labour force. We start with a base run where there are no labour market programmes, and gradually increase the flow into the programmes. Table 2 shows us what happens:

Table 2

γ	e_1	e_2	u_1	u_2	r	v	θ	w_1	w_2	TP_1	TP_2
0.0	80.00	10.00	6.00	4.00	0.00	2.07	0.259	100.00	100.00	100.00	100.00
0.001	81.72	7.87	6.54	3.36	0.50	1.91	0.232	99.97	100.72	99.71	100.28
0.002	82.98	6.40	6.92	2.85	0.85	1.81	0.217	99.96	101.22	99.57	100.54
0.005	85.14	4.02	7.52	1.89	1.42	1.67	0.197	99.97	102.02	99.47	101.08
0.01	86.65	2.45	7.91	1.19	1.79	1.59	0.186	99.98	102.56	99.49	101.52
0.02	87.73	1.37	8.17	0.68	2.04	1.54	0.180	99.99	102.94	99.54	101.87
0.05	88.53	0.59	8.36	0.30	2.22	1.50	0.176	100.01	103.21	99.61	102.13
0.1	88.83	0.30	8.42	0.15	2.29	1.49	0.175	100.02	103.31	99.63	102.23
0.5	89.09	0.06	8.48	0.03	2.34	1.48	0.174	100.02	103.40	99.66	102.32

We set $q = m\theta^{-0.4}$ and $k = w_a$. The following parameter values pertain to this table: A = 0.75051093; B = 0.69785026[3]; $\rho_r = 0.6$; $\rho_1 = \rho_2 = 0.5$; $y = 115$; $\delta = 0.05/365$; $\phi = 1/2400$; $\lambda = 1/1080$; $\pi = 1/150$; $\sigma = 1/900$; $m = 0.0125$. The initial values of w_1 and w_2 are 112.5 and 106.11, respectively.

e_1, e_2, u_1, u_2, r, and v, are all given as percentages of the labour force. w_1, w_2, TP_1, and TP_2, are all given as indices with the value in the base run set equal to 100.00.

As can be seen, the general effect of an increase in the flow into labour market programmes in this scenario is an increase in the primary labour force, a decrease in the secondary labour force, and a slight reduction in total unemployment. Total production decreases if $y_r = 0$, and increases if

[3] The values of the firm- and union-power exponentials in the Nash bargain have been chosen so as to set the unemployment and employment rates at exact values in the base runs. This allows simpler comparisons between the base run and the subsequent equilibrium to be made. This is true for all simulations.

$y_r = ay$. Whether labour market programmes are considered successful here largely depends on whether those on labour market programmes are productive or not.

4.2 Skill Loss and Lower Search Intensity. 100% Chance of Exiting into the Primary Labour Force from Secondary Employment; 50% Chance of Exiting into the Primary Labour Force from Labour Market Programmes.

In this section, we look at an example of a labour market which exhibits both skill loss and lower search intensity amongst those in the secondary labour force. We assume that those in secondary employment are eighty percent as productive as those in primary employment, i.e. a=0.8, and that those in secondary unemployment only search fifty percent as intensively as those in primary unemployment, i.e. c=0.5. A secondary worker in regular employment has a 100% chance of exiting into the primary labour force, whilst a secondary worker on a labour market programme has a fifty percent chance of exiting into the primary labour force. We start with a base run where there are no labour market programmes, and gradually increase the flow into the programmes. Table 3 shows us what happens:

Table 3

γ	e_1	e_2	u_1	u_2	r	v	θ	w_1	w_2	TP_1	TP_2
0.0	80.00	10.00	6.00	4.00	0.00	1.28	0.160	100.00	100.00	100.00	100.00
0.001	80.19	9.21	6.21	3.81	0.57	1.23	0.152	99.99	100.35	99.55	100.08
0.002	80.40	8.50	6.40	3.61	1.08	1.19	0.145	99.99	100.66	99.18	100.17
0.005	81.00	6.80	6.84	3.06	2.30	1.10	0.132	99.98	101.37	98.40	100.51
0.01	81.82	5.00	7.26	2.37	3.55	1.02	0.121	99.99	102.09	97.76	101.03
0.02	82.80	3.22	7.64	1.58	4.75	0.96	0.113	100.01	102.78	97.32	101.70
0.05	83.89	1.54	7.94	0.78	5.85	0.91	0.109	100.04	103.41	97.08	102.47
0.1	84.40	0.83	8.06	0.42	6.30	0.89	0.107	100.06	103.68	97.03	102.83
0.5	84.89	0.17	8.15	0.09	6.70	0.87	0.106	100.07	103.92	97.00	103.18

We set $q = m\theta^{-0.4}$ and $k = w_a$. The following parameter values pertain to this table: $A = 0.86462813$; $B = 0.808841$; $\rho_r = 0.6$; $\rho_1 = \rho_2 = 0.5$; $y = 115$; $\delta = 0.05/365$; $\phi = 1/1600$; $\lambda = 1/360$; $\pi = 1/150$; $\sigma = 1/600$; $m = 0.025$. The initial values of w_1 and w_2 are 113.5 and 87.31, respectively. e_1, e_2, u_1, u_2, r, and v, are all given as percentages of the labour force. w_1, w_2, TP_1, and TP_2, are all given as indices with the value in the base run set equal to 100.00.

Again, using labour market programmes leads to an increase in the primary labour force and a decrease of the secondary labour force. Total unemployment initially increases very slightly before declining with increased flows onto labour market programmes. As with the previous example, whether total production increases or decreases depends strongly on whether those on labour market programmes are productive or not.

4.3 Skill Loss and Lower Search Intensity. 50% Chance of Exiting from Secondary Employment into the Primary Labour Force; 100% Chance of Exiting from a Labour Market Programme into the Primary Labour Force.

In this section, the productivity and search intensity of secondary workers is the same as in the immediately preceding example. This time, however, secondary workers in regular employment have a fifty percent chance of exiting into the primary labour force whilst those on labour market programmes have a 100% chance of leaving the secondary labour force. Thus, here, labour market programmes are completely successful in transforming secondary workers into primary workers. This is in contrast to regular jobs for secondary workers, which give the worker a fifty percent chance of exiting into the primary labour force. Table 4 gives us the results:

Table 4

γ	e_1	e_2	u_1	u_2	r	v	θ	w_1	w_2	TP_1	TP_2
0.0	80.00	10.00	6.00	4.00	0.00	2.07	0.259	100.00	100.00	100.00	100.00
0.001	81.33	6.79	7.85	3.50	0.52	1.63	0.170	99.95	105.43	99.03	99.51
0.002	82.82	4.82	8.80	2.73	0.82	1.47	0.145	100.00	107.94	99.10	99.86
0.005	87.35	0.00	9.56	1.77	1.33	1.32	0.138	100.21	–	100.03	101.26
0.01	87.71	0.00	9.98	0.92	1.39	1.29	0.129	100.26	–	100.48	101.77
0.02	87.89	0.00	10.22	0.47	1.42	1.27	0.125	100.29	–	100.71	102.03
0.05	88.00	0.00	10.37	0.19	1.44	1.26	0.122	100.31	–	100.84	102.18
0.1	88.04	0.00	10.42	0.10	1.45	1.26	0.121	100.31	–	100.89	102.24
0.5	88.07	0.00	10.46	0.02	1.45	1.26	0.120	100.32	–	100.93	102.28

We set $q = m\theta^{-0.4}$ and $k = w_a$. The following parameter values pertain to this table: $A = 0.79652669$; $B = 0.68219587$; $\rho_r = 0.6$; $\rho_1 = \rho_2 = 0.5$; $y = 115$; $\delta = 0.05/365$; $\phi = 1/2400$; $\lambda = 1/1080$; $\pi = 1/150$; $\sigma = 1/900$; $m = 0.0125$. The initial values of w_1 and w_2 are 112.5 and 83.49, respectively. e_1, e_2, u_1, u_2, *r*, and *v*, are all given as percentages of the labour force. w_1, w_2, TP_1,and TP_2, are all given as indices with the value in the base run set equal to 100.00.

The consequence of increasing the flow into labour market programmes is to increase the proportion of workers in the primary labour force and to decrease the proportion of workers in the secondary labour force. Notice that when $\gamma = 0.005$ and above, secondary employment is completely crowded out. This is due to the fact that the worker in secondary unemployment has a higher expected lifetime income than if they were to enter secondary employment, i.e. $\Lambda_{u_2} > \Lambda_{e_2}$. The total unemployment rate is higher for all levels of labour market programme usage.

Looking at what happens to total production, we see that for low values of γ, total production is lower than in the base run. However at values of 0.005 and above it is higher, even when those on labour market programmes are completely unproductive. The increase of primary employment and the reduction in vacancies is more than enough to offset the loss of production from secondary employment.

4.4 Skill Loss and Lower Search Intensity. 50% Chance of Exiting from Secondary Employment into the Primary Labour Force; 100% Chance of Exiting from a Labour Market Programme into the Primary Labour Force.

In this section, we see a labour market similar to that analysed in Section 4.3 except that skill loss is even more pronounced. Secondary workers are 70% as productive as their primary counterparts, rather than 80% as in the previous example, i.e. *a*=0.7 here, rather than *a*=0.8. Table 5 on the following page shows us what happens.

Here, we see that by increasing the value of γ, we increase the proportion of those in the primary labour force whilst we decrease the proportion of those in the secondary labour force, just as in the previous examples. Total unemployment is higher when labour market programmes are used. Notice that secondary unemployment decreases when γ is increased to 0.001, increases when γ is increased to 0.002, but again continues to decrease when γ is increased still further. This erratic behaviour of secondary unemployment is due to the fact that when γ is increased to 0.001, it is still incentive compatible for the secondary worker in unemployment to accept a regular job. However, when γ is increased to 0.002 (and higher values), secondary employment is completely crowded out since $\Lambda_{u_2} > \Lambda_{e_2}$. As a result, there is a sudden halt to the flow from U_2 into E_2. Consequently secondary unemployment increases, despite the fact that there is an increased flow out of secondary unemployment into labour market programmes. When γ is increased still further, secondary unemployment again continues to fall since secondary employment cannot be crowded out any more.

Table 5

γ	e_1	e_2	u_1	u_2	r	v	θ	w_1	w_2	TP_1	TP_2
0.0	70.00	20.00	6.00	4.00	0.00	2.07	0.259	100.00	100.00	100.00	100.00
0.001	72.71	14.30	8.50	3.90	0.59	1.61	0.154	99.96	104.25	98.90	99.40
0.002	83.66	0.00	10.20	4.72	1.42	1.47	0.144	100.41	–	100.13	101.34
0.005	84.89	0.00	11.41	2.11	1.58	1.40	0.122	100.55	–	101.71	103.06
0.01	85.32	0.00	11.92	1.10	1.66	1.37	0.115	100.60	–	102.26	103.67
0.02	85.53	0.00	12.21	0.57	1.70	1.35	0.111	100.63	–	102.54	103.99
0.05	85.66	0.00	12.39	0.23	1.72	1.34	0.108	100.65	–	102.71	104.18
0.1	85.71	0.00	12.45	0.12	1.73	1.34	0.107	100.66	–	102.77	104.24
0.5	85.74	0.00	12.50	0.02	1.74	1.34	0.107	100.66	–	102.81	104.29

We set $q = m\theta^{-0.4}$ and $k = w_a$. The following parameter values pertain to this table: $A = 0.81075038$; $B = 0.73560319$; $\rho_r = 0.6$; $\rho_1 = \rho_2 = 0.5$; $y = 115$; $\delta = 0.05/365$; $\phi = 1/2100$; $\lambda = 1/1080$; $\pi = 1/150$; $\sigma = 1/1800$; $m = 0.0125$. The initial values of w_1 and w_2 are 112.0 and 77.21, respectively.

e_1, e_2, u_1, u_2, *r*, and *v*, are all given as percentages of the labour force. w_1, w_2, TP_1,and TP_2, are all given as indices with the value in the base run set equal to 100.00.

Notice that both measures of total production show an initial decline before increasing. Even if those on labour market programmes are completely unproductive, total production is higher than in the base run since the more-productive primary employment is crowded in at the expense of the less-productive secondary employment. Furthermore, vacancies decline meaning that less productive potential is spent on maintaining vacancies. Thus, even though total employment is lower when labour market programmes are in existence, the level of total production is still higher

4.5 Skill Loss but No Reduction in Search Intensity. 100% Chance of Exiting from Secondary Employment into the Primary Labour Force; 10% Chance of Exiting from a Labour Market Programme into the Primary Labour Force.

In this, our final example, there is skill loss present but no reduction in search intensity. Thus in this example, *a*=0.8 and *c*=1.0. Those secondary workers in regular employment have a 100% chance of exiting into the primary labour force, i.e. $\mu = 1.0$, whilst those on labour market programmes have only a 10% chance of exiting into the primary labour force, i.e. $\eta = 0.1$. Table 6 shows us what happens when labour market programmes are used:

Table 6

γ	e_1	e_2	u_1	u_2	r	v	θ	w_1	w_2	TP_1	TP_2
0.0	80.00	10.00	6.00	4.00	0.00	1.60	0.160	100.00	100.00	100.00	100.00
0.001	79.54	9.88	6.00	3.98	0.60	1.58	0.159	100.00	100.09	99.38	99.94
0.002	79.09	9.77	6.00	3.95	1.19	1.56	0.157	100.00	100.19	98.78	99.87
0.005	77.77	9.43	6.01	3.88	2.91	1.51	0.153	99.99	100.47	96.99	99.69
0.01	75.67	8.89	6.01	3.77	5.65	1.42	0.146	99.98	100.93	94.16	99.40
0.02	71.86	7.92	6.03	3.55	10.64	1.27	0.133	99.97	101.83	89.03	98.87
0.05	63.37	5.73	6.07	2.92	21.92	0.96	0.107	99.95	104.13	77.51	97.79
0.1	56.41	3.71	6.03	2.12	31.73	0.72	0.089	99.97	106.45	67.86	97.22
0.5	50.34	0.94	5.66	0.57	42.48	0.51	0.082	100.10	109.48	58.52	97.83

We set $q = m\theta^{-0.4}$ and $k = w_a$. The following parameter values pertain to this table: $A = 0.87091801$; $B = 0.70512695$; $\rho_r = 0.9$; $\rho_1 = \rho_2 = 0.5$; $y = 115$; $\delta = 0.05/365$; $\phi = 1/1600$; $\lambda = 1/180$; $\pi = 1/150$; $\sigma = 1/300$; $m = 0.025$. The initial values of w_1 and w_2 are 113.5 and 83.86, respectively.

e_1, e_2, u_1, u_2, *r*, and *v*, are all given as percentages of the labour force. w_1, w_2, TP_1,and TP_2, are all given as indices with the value in the base run set equal to 100.00.

Starting from an initial situation where total unemployment is 10%, we see that by increasing the number of workers on labour market programmes, we further reduce unemployment. However, whilst we reduce both primary and secondary unemployment, we also reduce both primary and secondary employment. Regular employment is significantly crowded out be labour market programmes. As a result of this crowding out, both measures of total production decline; TP_1 significantly.

This example shows that the usage of labour market programmes in this scenario is a failure. Regular employment declines, as does total production. Furthermore, unlike in all of the previous examples, the proportion of workers in the primary labour force declines. Thus the aim of transforming secondary workers into primary workers, fails.

5 Conclusion

From the preceding analysis it is painfully clear that the usage of labour market programmes in the model which we have specified in this paper is not an unmitigated success. Five examples of labour markets were given in Section 4; each of these indicated varying results when labour market programmes were directed at secondary workers in unemployment.

In four of the examples above, there was a possibility that total production in the economy could be increased by the usage of labour market programmes. However, this increase was not guaranteed in any of the examples. Total production tended to increase when skill loss existed amongst secondary workers and when those on labour market programmes were as productive as those in secondary employment. Examples 4.1 and 4.2 illustrate how whether those on labour market programmes are as productive as those in secondary employment or totally unproductive affects whether introducing labour market programmes has a positive or a negative effect on total production. In example 4.5, where labour market programmes crowd out all other states in the labour market, total production declines slightly when those on labour market programmes are as productive as those in secondary employment, but declines drastically when those on labour market programmes are unproductive. Thus whether total production increases or decreases when labour market programmes are introduced often depends on whether those on labour market programmes are productive or not.

In examples 4.3 and 4.4, we see situations when secondary employment is completely crowded out by the usage of labour market programmes. This is due to the fact that it is more-appealing for the secondary worker to be in unemployment rather than in employment, i.e. $\Lambda_{u_2} > \Lambda_{e_2}$. Despite this total crowding out of regular employment for secondary workers, total production increases, even if those on labour market programmes are completely unproductive. This is due to two things: Firstly, there is an increase in the number of workers in primary employment, where workers are fully productive; and, secondly, there is a reduction in the number of vacancies. Since maintaining vacancies involves using up a certain amount of the productive potential of the economy, any reduction in the number of vacancies increases the total production of the economy.

All in all, we find that the unqualified usage of labour market programmes in the context given, is not guaranteed to give wholly positive results. If a policy maker wishes to use labour market programmes in the manner suggested in this paper, then they must decide whether to do so in full knowledge of the parameter values of the model. Only then can the policy maker determine whether the effects of labour market programmes will be beneficial or otherwise.

6 Bibliography

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7 Appendix

Using equations [1], [2], [3], and [5], and the identity $1 \equiv e_1 + e_2 + u_1 + u_2 + r$, we gain the following:

$$\begin{bmatrix} \phi & 0 & -\alpha & 0 \\ \phi & \mu\sigma & -(\alpha+\lambda) & \eta\pi \\ 0 & \gamma\sigma & 0 & -c\alpha\pi \\ \gamma & \gamma & \gamma & (\gamma+\pi) \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ u_1 \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \gamma \end{bmatrix}$$

Solving for e_1, e_2, u_1, and r gives us:

[A1] $$e_1 = \frac{\alpha\pi\sigma(c\alpha\mu + \gamma\eta)}{V}$$

[A2] $$e_2 = \frac{c\alpha\phi\lambda\pi}{V}$$

[A3] $$u_1 = \frac{\phi\pi\sigma(c\alpha\mu + \gamma\eta)}{V}$$

[A4] $$r = \frac{\phi\gamma\lambda\sigma}{V}$$

where $V \equiv c\alpha\phi\lambda\pi + \phi\gamma\eta\pi\sigma + c\alpha\phi\mu\pi\sigma + \phi\lambda\sigma(\gamma + \pi) + c\alpha^2\mu\pi\sigma + \alpha\gamma\eta\pi\sigma$

And invoking the identity $1 \equiv e_1 + e_2 + u_1 + u_2 + r$, we gain the expression for u_2:

[A5] $\quad u_2 = \dfrac{\phi\lambda\pi\sigma}{V}$

Differentiating [A1] with respect to γ, we find that

$$\left.\frac{\partial e_1}{\partial \gamma}\right|_{\theta=\bar{\theta}} = \frac{\alpha\pi\sigma}{V^2}\begin{bmatrix} \eta\left\{c\alpha\phi\lambda\pi + \phi\gamma\eta\pi\sigma + c\alpha\phi\mu\pi\sigma + \phi\lambda\sigma(\gamma+\pi) + c\alpha^2\mu\pi\sigma + \alpha\gamma\eta\pi\sigma\right\} \\ -\sigma(c\alpha\mu + \gamma\eta)(\phi\eta\pi + \phi\lambda + \alpha\eta\pi) \end{bmatrix}$$

If we set $\eta = 0$ and $\mu = 1$, then $\left(\partial e_1/\partial\gamma\right)\big|_{\theta=\bar{\theta}} < 0$. However, if we set $\eta = 1$ and μ sufficiently low enough, then $\left(\partial e_1/\partial\gamma\right)\big|_{\theta=\bar{\theta}} > 0$. Since nothing precludes this differential from being equal to zero, we have the following: $\boxed{\left.\dfrac{\partial e_1}{\partial\gamma}\right|_{\theta=\bar{\theta}} \gtreqless 0}$

Differentiating [A2] with respect to γ, yields the following:

$$\boxed{\left.\frac{\partial e_2}{\partial\gamma}\right|_{\theta=\bar{\theta}} = -e_2\,\frac{\sigma(\phi\eta\pi + \phi\lambda + \alpha\eta\pi)}{V} < 0}$$

Differentiating [A3] with respect to γ yields

$$\left.\frac{\partial u_1}{\partial \gamma}\right|_{\theta=\bar{\theta}} = \frac{\phi\pi\sigma}{V^2}\begin{bmatrix} \eta\left\{c\alpha\phi\lambda\pi + \phi\gamma\eta\pi\sigma + c\alpha\phi\mu\pi\sigma + \phi\lambda\sigma(\gamma+\pi) + c\alpha^2\mu\pi\sigma + \alpha\gamma\eta\pi\sigma\right\} \\ -\sigma(c\alpha\mu + \gamma\eta)(\phi\eta\pi + \phi\lambda + \alpha\eta\pi) \end{bmatrix}$$

When $\eta = 0$, we find that $\left(\partial u_1/\partial\gamma\right)\big|_{\theta=\bar{\theta}} < 0$. However, when $\eta = 1$ and μ is low enough, we find that $\left(\partial u_1/\partial\gamma\right)\big|_{\theta=\bar{\theta}} > 0$. Since nothing precludes this differential from being equal to zero, we have the following:

$$\boxed{\left.\frac{\partial u_1}{\partial\gamma}\right|_{\theta=\bar{\theta}} \gtreqless 0}$$

Differentiating [A5] with respect to γ yields

$$\boxed{\left.\frac{\partial u_2}{\partial \gamma}\right|_{\theta=\bar{\theta}} = -u_2 \frac{\sigma[\phi\eta\pi + \phi\lambda + \alpha\eta\pi]}{V} < 0}$$

Differentiating [A4] with respect to γ yields

$$\boxed{\left.\frac{\partial r}{\partial \gamma}\right|_{\theta=\bar{\theta}} = \frac{\phi\lambda\pi\sigma}{V^2}\left(c\alpha\phi\lambda + c\alpha\phi\mu\sigma + \phi\lambda\sigma + c\alpha^2\mu\sigma\right) > 0}$$

Since $\alpha'(\theta) > 0$, this implies that $\operatorname{sign}(\partial\text{stock}/\partial\alpha) = \operatorname{sign}(\partial\text{stock}/\partial\theta)$. Therefore, if we can find the differential of a labour market stock with respect to α, then we can tell from its sign whether that stock will increase with an increase in θ or otherwise.

Differentiating [A1] with respect to α gives us

$$\boxed{\frac{\partial e_1}{\partial \alpha} = \frac{\phi\pi\sigma}{V^2}\begin{bmatrix} c\alpha\mu\{c\alpha\lambda\pi + 2\gamma\eta\pi\sigma + c\alpha\mu\pi\sigma + 2\lambda\sigma(\gamma+\pi)\} \\ +\gamma\eta\sigma\{\gamma\eta\pi + \lambda(\gamma+\pi)\} \end{bmatrix} > 0}$$

Differentiating [A2] with respect to α yields the following:

$$\boxed{\frac{\partial e_2}{\partial \alpha} = e_2 \frac{\sigma}{V}\left[\frac{\phi\gamma\eta\pi}{\alpha} + \frac{\phi\lambda(\gamma+\pi)}{\alpha} - c\alpha\mu\pi\right] \gtreqless 0}$$

Differentiating [A3] with respect to α gives us

$$\frac{\partial u_1}{\partial \alpha} = \frac{\phi\pi\sigma}{V^2}\begin{bmatrix} c\mu\sigma\{\phi\lambda(\gamma+\pi) - c\alpha^2\mu\pi - 2\alpha\gamma\eta\pi\} \\ -\gamma\eta\pi(c\phi\lambda + \gamma\eta\sigma) \end{bmatrix}$$

If we set $\eta = 0$, and μ sufficiently low, then $\partial u_1/\partial\alpha > 0$. However, if we have $\eta = 1$ and μ sufficiently low then $\partial u_1/\partial\alpha < 0$. Since nothing precludes this differential from being equal to zero, we have the following:

$$\boxed{\frac{\partial u_1}{\partial \alpha} \gtrless 0}$$

Differentiating [A5] with respect to α yields

$$\boxed{\frac{\partial u_2}{\partial \alpha} = -\frac{u_2}{V}\left[c\phi\lambda\pi + c\phi\mu\pi\sigma + 2c\alpha\mu\pi\sigma + \gamma\eta\pi\sigma\right] < 0}$$

Differentiating [A4] with respect to α yields

$$\boxed{\frac{\partial r}{\partial \alpha} = -\frac{r}{V}\left[c\phi\lambda\pi + c\phi\mu\pi\sigma + 2c\alpha\mu\pi\sigma + \gamma\eta\pi\sigma\right] < 0}$$